a Build reusable skills

Faithful statements

Gaussian bounds
Lax–Milgram
Wave energy
…

→ proofs

Reusable skill

When Conditions
How Method
What Guarantee
Why Evidence

b Connect the statements

Gaussian bounds + Lax–Milgram
Compare energies; control residual error.
→ Precondition the solve

Wave energy conservation
Combine energy and matrix residual bounds.
→ Bound prediction error at all times

Preserved: objects · assumptions · quantifiers · norms · randomness · guarantees

c Apply to two computational tasks

Steady heat flow: solve accurately

PROBLEM

Find the steady temperature.
Zero ends; a distributed heat source.

Conductivity contrast: 100 / 1,000
8,192 intervals · 512 coefficients
Relative energy-norm error: 10^{-6}

SKILL · PRECONDITION AND VERIFY

2,048 sketch rows · 1% failure budget

G: 8,192 × 512 **SG: 2,048 × 512** **R: 512 × 512**

Check the original equation's residual.

RESULT · Condition numbers

Contrast	Reference	Gaussian
100	99.6	**8.83–8.95**
1,000	964.0	**8.83–8.94**

All 6 runs: 22 updates · error < 10^{-6}

Wave responses: compress and reuse

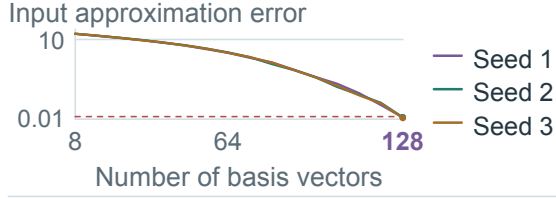
PROBLEM

Predict a fixed-end string's motion.
384 displacement / velocity inputs.

128 spatial modes · 256 coordinates
Input amplitude norm ≤ 1
Absolute energy-norm error: 0.01

SKILL · LEARN A RESPONSE BASIS

Randomly combine → orthogonalize



RESULT · A smaller response library

384 inputs → 128 basis responses

Full library **24 MiB**
Compressed **8.375 MiB**

Energy conserved → all-time error ≤ 0.01